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**CS435**

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*Lab#12*

***Group 1***

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1. Problem 1

Yes, the graph has a Hamiltonian cycle. A spanning simple cycle is shown below:

1. Problem 2

* Given the graph shown (top), *G = (V, E)*, which has 6 vertices and 6 edges.
  + - *G* is also a subgraph of *K*6, the complete graph having 15 edges (bottom graph).
* We obtain an instance *(H, c, k)* of the TSP, such that:
  + - *H = K*6 (bottom graph).
    - *k =* 0
* We note that defining *(H, c, k)* from G can be done in polynomial time.
* We need to show that :
  + - If and only if *(H, c, k)* has a Hamiltonian cycle with , then *G* has a Hamiltonian cycle.
    - If *G* has a Hamiltonian cycle *C*, then *C* is Hamiltonian in *H*, too.
* Note that *G* has a Hamiltonian cycle *C = A – B – E – F – D – C – A.* Also, *C* is a Hamiltonian cycle in *H* because every vertex in *H* is in *C* and *C* has no cycles.
* Note that each edge *e* of *C* is in *G* (i.e., for all edges of *C*).

for all edges in *C*, i.e.

From (1) we conclude that a solution to the HC problem having input *G = (V, E)* gives rise to a solution to the TSP problem we defined from *G* with input data *(H, c, k)*.

* Notice that the cycle *C' = A – B – E – F – D – C – A* is a Hamiltonian cycle in *H* and the sum of its edge weights is 0 (which is the value of *k*).
* By definition of *c(e)*, we know that edge weights are zero only if they belong to *E*, and since the weights of edges in *C’* are all equal to 0 (because their sum is zero), then *C’* is a Hamiltonian cycle in *G*. → (2)
* From (1) and (2) we conclude that the Hamiltonian cycle problem is polynomial reducible to the TSP problem.

1. Problem 3

To show that TSP is NP-complete we need to prove the following:

1. TSP is NP-problem:

A solution to the TSP can be verified using the following algorithm:

|  |  |
| --- | --- |
| **Algorithm** Verify\_TSP\_Solution | *Running time* |
| ***Input:*** Graph G = V, E, a path C, a non-negative constant k |  |
| ***Output:*** *true* if C is a cycle and sum of its edges cost is ≤ k , *false* otherwise |  |
| for every vertex vC in VC do |  |
| calculate deg(vC) | *O(m.n)* |
| if (deg(vC) != 2) return *false* |  |
| perform BFS on C, calculate numberOfComponents and sumOfEdgeCost | *O(m + n)* |
| if (numberOfComponents != 1 || sumOfEdgeCost > k) then return *false* | *O(1)* |
| return *true* | *O(1)* |

⸫ Running time is O(m.n) = O(n3) → Polynomial → TSP is NP

1. Since the Hamiltonian cycle problem (which is an NP-problem) is polynomial reducible to TSP, and from No. 1 above: we conclude that the TSP is NP-complete.
2. Problem 4

Consider the graph, G below:

Graph G = {AB, KD, EB, BD, DF, BJ, DL}.

1. The smallest vertex cover for G is U = {B, D}, ⸫ s = |U| = 2.
2. We apply the VertexCoverApprox algorithm to it as follows:

C = new empty set

G still has edges

⸫ select edge AB

add vertices A, B to C → C = {A, B}

remove edges incident to A or B from G → G = {KD, DF, DL}

G still has edges

⸫ select edge KD

add vertices K, D to C → C = {A, B, K, D}

remove edges incident to K or D → G = {}

G has no edges

return C = {A, B, K, D}

Notice that |C| = 4 = 2 \* |U| = 2\*s.

⸫ Applying the VertexCoverApprox algorithm results in size = 2\*s.